Local Polynomial Estimate of Surface Laplacian

Kongming Wang* and Henri Begleiter*

Summary: This paper describes a method for estimating the surface Laplacian of brain potentials. The method consists of two steps: local surface approximation by its tangent plane and local polynomial fitting. Compared to previous methods for estimating surface Laplacian, this method has some new features. First, it can estimate the surface Laplacian at any point of the scalp, including the locations of the peripheral electrodes. Secondly, it estimates the brain potential and the surface Laplacian at any point simultaneously. This reduces the risk of error propagation, which occurs when the brain potential is interpolated first and the surface Laplacian is then computed based on the interpolated brain potential. Finally, the method automatically adapts to noisy data by using more or less measurements at neighboring electrodes based on estimated noise level. Simulations suggest that this method is effective. Application to event-related potentials are also presented.

Key words: Event-related potentials; Local polynomial fitting; Surface Laplacian.

Introduction

Electric currents generated by sources in brain are volume conducted through brain, cerebrospinal fluid, skull, and scalp to the recording electrodes. Therefore, brain potentials measured at an electrode represent the summation of signals from many sources over the brain. It has been shown that surface Laplacian measures provide reference-independent estimates of radial current source density (Katznelson 1981; Le et al. 1994; Nunez et al. 1994) and eliminate the common activity due to volume conduction from distant sources (Nunez 1995). Srinirasan et al. (1998) show that the surface Laplacian estimate of cortical surface potentials reduce the artificial coherence in individual frequency bands due to volume conduction.

There exist many implementations for computing surface Laplacian of brain potential. The so-called local methods use only the potentials at "nearest neighbor" electrodes and the surface Laplacian is computed by a finite difference scheme (Hjorth 1975; Katznelson 1981) or by a least squares solution of fitting a local quadratic representation of the potentials (Le et al. 1994). Since the electrodes are not placed at regular positions, these local methods interpolate potentials at a regular grid. Therefore, the surface Laplacian is computed based on interpolated potentials and it is very sensitive to noise, because the Laplacian operation amplifies high spatial frequencies (Le et al. 1994). Since a peripheral location of the recording montage is not surrounded by electrodes, the surface Laplacian at the peripheral location can not be estimated by these local methods. The so-called global methods construct a global potential function and then compute the surface Laplacian of the global potential function. So the Laplacian at any point depends on the potentials at all electrodes. A popular interpolation function is the spherical spline (Perrin at al. 1987; Perrin at al. 1989). The global approach sometimes produces a distorted Laplacian estimate (Biggins et al. 1991; Fein et al. 1991; Le et al. 1994) and direct differentiation on the interpolated function may not be a consistent estimate of the Laplacian. Babiloni et al. (1995) evaluated the performances of these local and global methods. They found that the best surface Laplacian estimates were computed by second order spline including λ correction (smoothing parameter).

This paper presents a new implementation. It aims (1) to be a local method which assigns larger weight for closer electrodes; (2) to be able to estimate surface Laplacian at peripheral locations; (3) to estimate potential and surface Laplacian simultaneously so that direct differentiation on an interpolated function is unnecessary; and (4) to be robust to noise.

The surface Laplacian is independent of the head volume conductor model, though it depends on the

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shape of the scalp surface. We will assume a spherical scalp surface in this paper for the convenience of computing the true surface Laplacian in simulation studies. At any location on the scalp surface, the local surface is approximated by the tangent plane at this position. The locations of the neighboring electrodes are projected to this tangent plane and the surface Laplacian at the scalp location is approximated by the surface Laplacian on the tangent plane. The number of neighboring electrodes used in this computation depends on the noise level which can be estimated from the potentials recorded at all electrodes. Higher noise level requires more measurements from neighboring electrodes. Once the number of neighboring electrodes is determined and the projection of the locations of these electrodes on the tangent plane are computed, a quadratic fitting is solved by least squares method to estimate both potential and surface Laplacian at the scalp location.

We evaluate the performances of this method with simulations. It shows that this method performs better than the second order spline including λ correction in the case of large SNR, due to the fact that surface Laplacian is a local property of the scalp potentials. The second order spline estimate is better in the case of small SNR, since more data are needed for the estimate at any location in order to reduce the effect of noise. Hence a global method would be better. In practice, we would suggest the use of the second order spline estimate for noisy data such as single trials, and the local polynomial fitting for clean data such as the average of the single trials. The simulations also show that our adaptive method is robust to noise, meaning that its performance is not far away from the best that the local quadratic fitting can offer. For the second order spline estimate, the simulations indicate that a fixed small λ (say $\lambda = 0.1$) would work well for both small and large SNR cases.

Method

The scalp surface can be parametrized by two independent parameters and the surface Laplacian can be computed by the standard Laplacian in a parametric space. The polar coordinates are often used for parametrizing spherical surface, but they distort the relative positions of the electrodes (figure 1) and it will produce a distorted surface Laplacian. The projection of neighboring electrodes on the tangent plane at a scalp location reserves the relative positions of the electrodes very well. Therefore, we use the tangent plane to approximate the scalp surface locally. Figure 1 illustrates the tangent plane at electrode F1 and the projection of the neighboring electrodes on the tangent plane. The local coordinates of the projected neighboring electrodes on the tangent plane are derived in the Appendix.

Now we estimate the potential and the surface Laplacian at a scalp location (x,y,z). Note that whether (x,y,z) is an electrode position makes no difference. If K neighboring electrodes will be used to estimate the potential and the surface Laplacian at (x,y,z), then denote the local coordinates of the projected electrodes by (u_i, v_i), $i=1,...,K$. The local coordinate of (x,y,z) is $(0,0)$. We approximate the local potential surface (not scalp surface!) by a quadratic function

$$P(u,v) = a_0 + a_1 u + a_2 v + \frac{a_3}{2} u^2 + a_4 uv + \frac{a_5}{2} v^2$$

Then the potential at (x,y,z) is estimated by a_0 and the surface Laplacian at (x,y,z) is estimated by a_3+a_5, because the point (x,y,z) has local coordinates $(0,0)$. The coefficients $a = (a_0,...,a_5)$ are estimated by the least squares solution of the minimization problem

$$\min_a \sum_{i=1}^{K} W(u_i, v_i, h)(P_i - P(u_i, v_i))^2.$$

Here P_i is the measured potential at the scalp location with local coordinates (u_i, v_i), and $W(u_i, v_i, h)$ is a weight function which assigns larger weight for electrodes closer to (x,y,z). In particular, we will use the Epanechnikov weight function

$$W(u_i, v_i, h) = \frac{2}{\pi}\left(1 - \frac{u_i^2 + v_i^2}{h^2}\right)$$

if $u_i^2 + v_i^2 < h^2$, and $W(u_i, v_i, h) = 0$ if $u_i^2 + v_i^2 \geq h^2$. The bandwidth h is determined by the estimated noise level and will be discussed at the end of this section. The Epanechnikov weight function W is optimal in the sense of asymptotic mean squared error (Fan et al. 1997).

Estimating the noise level

The noise level is estimated by the difference between the measured potentials at the electrode positions and the linearly interpolated potentials using the measurements at neighboring electrodes. The linearly interpolated potential F_i at the location of electrode i is computed as follows. Project the three closest electrode locations onto the tangent plane at the location of electrode i and denote their local coordinates by (u_j, v_j), $j=1,2,3$. Then the linear interpolation of potential at electrode i based on the potentials at the three closest electrodes is given by $F_i = b_1$, where

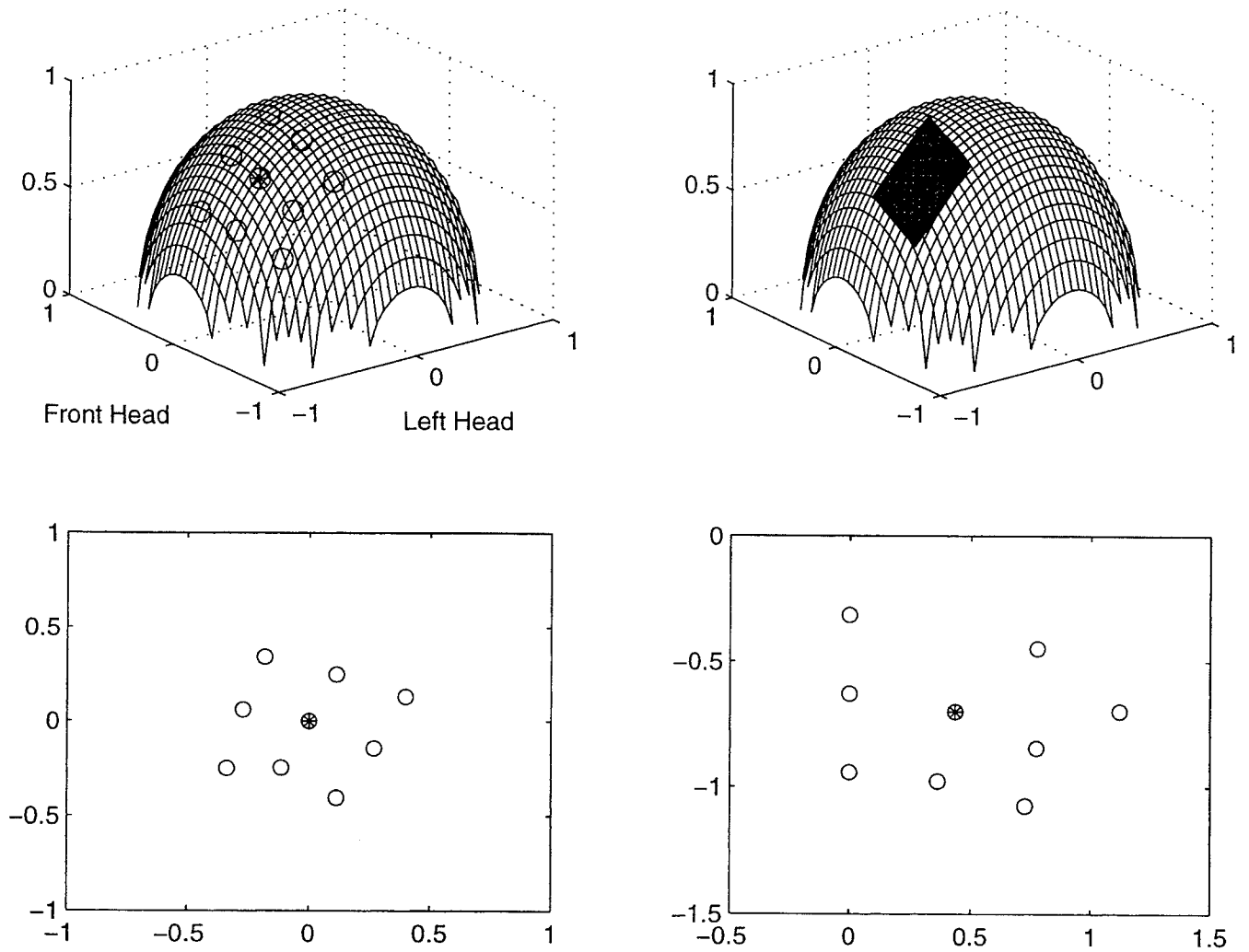

Figure 1. Projection of neighboring electrodes on the tangent plane at electrode F1. (a-top left) Spherical scalp surface and the locations of 9 electrodes. The location of F1 is marked with "*". (b-top right) The tangent plane at F1. (c-bottom left) The projection of the 9 locations on the tangent plane and their local coordinates. (d-bottom right) The polar parametrization of the 9 locations.

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 1 & u_1 & v_1 \\ 1 & u_2 & v_2 \\ 1 & u_3 & v_3 \end{pmatrix}^{-1} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix}.$$

The noise level is estimated by

$$\sigma = \frac{1}{2}\left[\sqrt{\frac{\sum_{i=1}^{N}(P_i - F_i)^2}{N-1}} + \text{median}\left\{|P_i - F_i|, i = 1,\ldots,N\right\}\right]$$

where N is the number of electrodes. Our simulations indicate that the first term overestimates the noise level while the second term underestimates the noise level. So we take their average as the estimate of the noise level.

This simple estimate is based on the assumption that the potential surface is smooth due to the volume conduction of the brain.

Determining the number of neighboring electrodes and the bandwidth *h*

First we determine how many measurements from neighboring electrodes should be used when estimating the potential and the surface Laplacian at a scalp location. We start from clean data ($\sigma = 0$). Simulation with different potential surfaces shows that the estimation with 11 measurements at neighboring electrodes produces almost optimal estimates for clean data if 61 electrodes are used for data recording. This is consistent with the method of Le et al. (1994) where 12 neighboring measure-

ments are used. For estimating the second derivatives of a regression function, the optimal bandwidth is proportional to $\sigma^{2/9}$ (Gasser and Müller 1984). Therefore, we determine the number of neighboring electrodes by K=11 if the noise level $\sigma \leq 0.1$, and

$$K = \text{round}\left[11\left(\frac{\sigma}{0.1}\right)^{2/9}\right] \quad (1)$$

if the noise level $\sigma > 0.1$. Here the function round () rounds a number to its closest integer. This means that 11 neighboring electrodes are used when data is almost clean ($\sigma \leq 0.1$), and more neighboring electrodes are used when the noise level increases.

If the number of recording electrodes is not 61, the same formula can be used to compute the number K of neighboring electrodes. The difference is that the number of neighboring electrodes for clean data is now a different number from 11, and it can be determined by simulation.

When the number K of neighboring electrodes is determined, the bandwidth h is then chosen as follows. Project the locations of the closest K+1 neighboring electrodes to the tangent plane at (x,y,z) and denote their local coordinates by (u_i, v_i), i=1,..., K+1. Let

$$d_i = \sqrt{u_i^2 + v_i^2}$$

and assume $d_i \leq d_{i+1}$ without loss of generality. The bandwidth h is defined by

$$h = (d_K + d_{K+1})/2.$$

Simulations

The new method is tested with two functions. The true surface Laplacian is computed through the polar parametrization of the scalp surface: $x = \cos(\theta)\sin(\phi)$, $y = \sin(\theta)\sin(\phi)$, $z = \cos(\phi)$, $\theta \in [0, 2\pi]$ and $\phi \in [0,\pi]$.

The clean data are computed from a test function $f(x,y,z)$ at 61 electrode locations. The first test function is defined by

$$f(x,y,z) = 5\,(\sin(\pi x)\cos(\pi y)\sin(\pi z) + 2z^2).$$

The second test function is defined by

$$f(x,y,z) = \sum_{i=1}^{5}\sum_{j=1}^{3} Q_{ij} L_i\left(u_j x + v_j y + w_j z\right)$$

where L_i is the Legendre polynomial of order i. The three points (u_j, v_j, w_j) are defined by $(\cos(\theta_j)\sin(\pi/4), \sin(\theta_j)\sin(\pi/4), \cos(\pi/4))$, θ_1=0, θ_2=2π/3, θ_3=4π/3. The coefficients Q_{ij} are generated random numbers:

$$Q = (Q_{ij}) = \begin{pmatrix} -2.1628 & 5.9546 & -0.9335 \\ -8.3279 & 5.9458 & 3.6290 \\ 0.6267 & -0.1882 & -2.9416 \\ 1.4384 & 1.6365 & 10.9159 \\ -5.7324 & 0.8732 & -0.6820 \end{pmatrix}$$

The second test function is an approximation to the theoretical potentials with three dipole sources, and it is in favour of the spline method which approximates a potential surface by a combination of the Legendre polynomials.

The noisy potential data are obtained by adding scaled white noise to the simulated potentials to give a prespecified SNR. The scaling factor is of course the noise level.

The simulation is carried out at five SNR levels. For each prespecified SNR, the simulation is performed with 50 replications. For each replication, we generate 61 normal random numbers and scale them so that the noisy data have the prespecified SNR. Then we estimate the surface Laplacian with four procedures: (I) the second order spline method with optimal λ correction, chosen by a grid search $\lambda \in \{0.1t: t=0,1,2,...,10\}$; (II) the second order spline method without λ correction; (III) the local quadratic fitting with optimal K (the number of neighboring electrodes), chosen again by a grid search $K \in \{11+2t: t=0,1,2,...,10\}$; (IV) the local quadratic fitting with K selected by the data-driven formula (1). The second order spline implementation is taken from Babiloni et al. (1995).

Table I gives the results of estimating the noise level. The estimation is good in most cases and it is poor when the data are clean, since the simple method overestimates the noise level. The column titled with "estimated σ" is the average of the estimated σ in 50 replications, and the relative errors are computed by

$$\text{error} = 100\frac{(\text{estimated } \sigma) - \sigma}{\sigma}\% \quad (2)$$

Table II gives the results of estimating the surface Laplacian of the first test function. The errors are computed as the ratio between the mean squared error (MSE) and the signal power (SP). Let $\hat{f}(i)$ be the estimate of the function value $f(i)$ (potential or surface Laplacian) at scalp location i. Assume that the estimates are computed at N locations on the scalp, then

Table I. The error of estimating the noise level with 61 electrodes, computed by using (2).

First Test Function				Second Test Function			
SNR	σ	estimated σ	error (%)	SNR	σ	estimated σ	error (%)
1	5.1482	5.4803	6.4512	1	9.1394	9.7328	6.4926
5	2.2609	2.5349	12.1230	5	4.0136	4.5315	12.9030
10	1.6402	1.8211	11.0280	10	2.9117	3.3616	15.4516
15	1.3555	1.5427	13.8056	15	2.3598	2.9309	24.2039
100	0.5117	0.9450	84.6910	100	0.8975	1.8388	104.8885

$$\mathrm{MSE} = \frac{1}{N}\sum_{i=1}^{N}\left(\hat{f}(i) - f(i)\right)^2, \qquad \mathrm{SP} = \frac{1}{N}\sum_{i=1}^{N}\left(f(i)\right)^2.$$

Let AMSE = $\Sigma_{j=1}^{50}$ MSE$_j$/50 be the average of the MSEs of the 50 replications. We define the estimation error by

$$\text{error} = 100\frac{\text{AMSE}}{\text{SP}}\% \qquad (3)$$

This relative error measurement is more informative than the AMSE.

Table III is the same as table II, except that it is computed with the second test function. The poor performance of the second order spline method without λ correction is partially due to the numerical instability (singular matrix inversion) of the method. This and the noise amplification of the Laplacian operation (Le et al. 1994) make it almost impossible to estimate the surface Laplacian from noisy data without smoothing (the λ correction).

Tables II and III show that the second order spline with λ correction is better for poor SNR, because the local quadratic fitting can not effectively reduce the variance of the estimator. For moderate SNR (10, 15 in our simulation), both methods are competitive. For clean data, the local method is better because the bias of the local estimator is smaller. The tables also show that formula (1), the method of determining the number of neighboring electrodes according to the estimated noise level, works well. For the λ correction, it seems that a small λ (say 0.1) works well for all SNR. Figure 2 plots a typical run of the simulations with SNR=10 and the second test function.

Table IV reports the simulation results with 105 electrodes and the second test function. The error of estimating the potentials is also included. We still use formula (1) to estimate the number of neighboring electrodes, even though a modified formula should be used because of more than 61 electrodes. The local quadratic fitting is competitive or better when SNR ≥ 15 for estimating surface Laplacian, and it is better when SNR ≥ 5 for estimating potentials. The second order spline with λ correction is the best among the previous surface Laplacian estimates with more than 64 electrodes (Babiloni et al. 1995).

Two reasons can explain why the local method uses many neighboring electrodes to estimate the potentials and the surface Laplacian at a scalp location (tables II and III). First, it needs many data points in order to suppress noise. Secondly, the weight function W implies that the fitting at many of those neighboring electrodes does not

Table II. Simulation results with the first test function and 61 electrodes. The errors of estimating surface Laplacian are computed by using (3) for all methods I - IV.

SNR	I		II	III		IV	
	error (%)	λ	error (%)	error (%)	K	error (%)	estimated K
1	79.0548	0.16	8764.218	148.1100	30.48	167.6808	26.22
5	70.7034	0.10	1726.072	95.3647	22.84	98.1511	22.00
10	68.4526	0.10	877.831	76.6997	19.20	78.6989	20.50
15	68.7859	0.10	581.779	69.3542	17.64	71.7324	19.70
100	67.6271	0.10	114.352	43.3342	11.08	56.5995	17.78

Table III. Simulation results with the second test function and 61 electrodes. The errors of estimating surface Laplacian are computed by using (3) for all methods I - IV.

SNR	I		II	III		IV	
	error (%)	λ	error (%)	error (%)	K	error (%)	estimated K
1	61.6382	0.108	7584.407	112.1548	30.8400	116.3906	29.84
5	53.8809	0.100	1492.176	64.3853	27.4800	67.0795	25.14
10	53.2761	0.100	761.059	54.6037	23.6800	56.8277	23.50
15	52.2033	0.100	529.694	52.9425	20.4800	54.9267	22.80
100	51.8548	0.100	97.604	36.2822	13.4400	44.0454	20.50

count much. The fitting at few closest neighboring electrodes count the most.

Application to visual P3 data

In this visual P3 experiment, subjects are presented with 280 visual stimuli with a uniform inter-stimulus interval of 1.6 seconds. There are 210 non-target stimuli in the shape of an outline of a square, 35 target stimuli in the shape of an X, and 35 novel stimuli, each a different colored polygon or other geometrical figure. The three different stimuli are presented in a random sequence. Subjects are instructed to respond to target stimulus by pressing a button. The visual P3 data are recorded from 64 channels (61 channels of International 10/20 system, two channels for monitoring eye movements, and the nose channel for reference). So the data to be analyzed have 61 channels. The sampling interval is 3.906 msec and 205 sample points (800 msec signal) after each stimulus are recorded for each condition. After amplification by a factor 10000, artifact threshold is set at 73.3 microvolts. Any trial with a value above the threshold is rejected, and the other trials are averaged to obtain the average ERP with improved SNR. The averages are plotted in figure 3 for a control subject and a alcoholic subject.

We will estimate the potential maps and the surface Laplacian at 400 msec using the proposed local quadratic fitting. For the control subject, the estimated noise level at 400 msec is $\sigma = 0.7545$, the estimated SNR is 172 (clean data), and the surface Laplacian is estimated with K=17 neighboring electrodes. For the alcoholic subject, the estimated noise level is $\sigma = 0.6290$, the estimated SNR is 211 (clean data), and the surface Laplacian is computed with K=16 neighboring electrodes. The simulations of the last section indicate that we should get accurate estimates for both potentials and surface Laplacian from such clean data, due to the averaging over trials. The estimated potentials and the estimated surface Laplacian are plotted in figure 4.

It can be seen from figure 4 that the surface Laplacian maps are much shaper than the potential maps. Both potential maps have similar differentiation from back to front, but the surface Laplacian maps indicate completely different current source densities. For the control subject, there are two major source areas (back-left and back-right) and one major sink area (front head). For the alcoholic subject, there is one major source area (back-left) and one major sink area (top of head).

Table IV. Simulation results with the second test function and 105 electrodes. The errors are computed by using (3). The columns 2, 4, and 6 are the errors of estimating surface Laplacian using the three procedures (I), (III), and (IV) respectively. The columns 8-10 are the errors of estimating potentials using the three procedures.

SNR	I		III		IV		error (%)		
	error (%)	λ	error (%)	K	error (%)	estimated K	I	III	IV
1	57.24	0.12	163.75	31.00	163.66	31.18	14.25	22.03	21.94
5	47.86	0.10	68.58	31.00	80.69	26.00	9.70	5.08	5.78
10	44.68	0.10	55.69	30.32	63.47	24.16	8.72	2.58	3.03
15	44.55	0.10	47.99	28.48	52.35	23.24	8.71	1.98	2.24
100	43.72	0.10	36.02	19.56	36.17	19.84	8.41	0.52	0.52

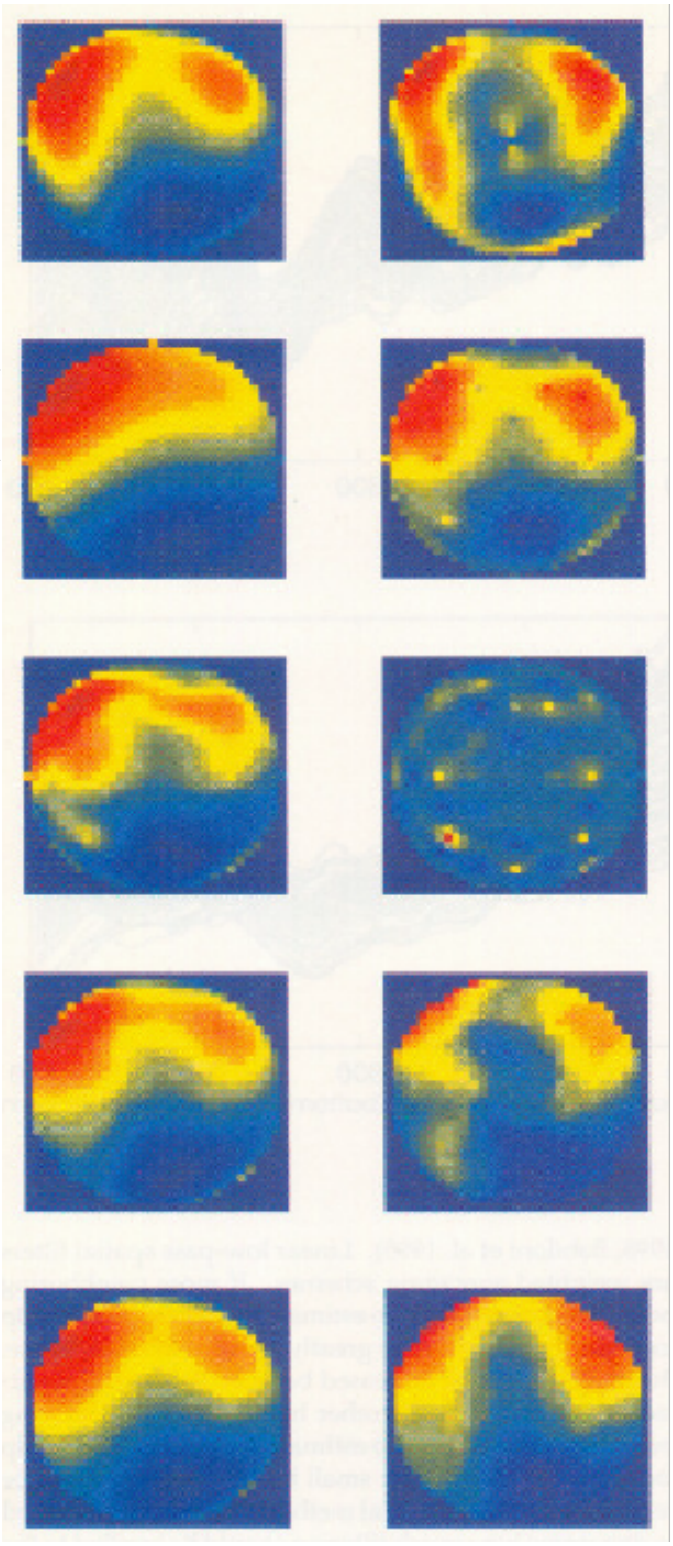

Figure 2. Plot of a run in the simulation with SNR=10 and the second test function. The left column plots the scalp potentials and the right column plots the surface Laplacian. The rows from top to bottom: true data (first row); estimates by method I (second row); estimates by method II (third row); estimates by method III (forth row); estimates by method IV (fifth row). All images are plotted from the view of top head.

Discussion

Many techniques have been employed to obtain high spatial resolution EEG and ERP. Increasing the number of electrodes will improve the resolution, but its usefulness is limited by the distortion of the volume conduction through brain, skull, and scalp (Gevins 1996). Methods for reducing this distortion can be model-dependent and model-independent. The model-dependent methods such as cortical imaging and source localization (linear estimation) are usually computationally intensive, while model-independent methods such as surface Laplacian are usually computationally simple.

Cortical imaging is for approximating potential fields on the cortical surface (Marin et al. 1998; Babiloni et al. 1997;Sidman et al. 1996). It uses equivalent current dipole layers as a source model to fit the recorded scalp potentials, and then computes the cortical potentials from the fitted sources. This method is based on a head and volume conduction model, and it is reference-electrode dependent. Realistic head models other than a three concentric sphere have been used in recent researchs (Marin et al. 1998; Babiloni et al. 1997; Fuchs et al. 1998a, 1998b).

Source localization techniques are designed to locate the source distributions in the brain based on the recorded scalp EEG or ERP (Koles 1998; Mosher and Leahy 1998; Awada et al. 1998). These methods are for locating 3-D source distributions (cortical and deep sources). Again head models range from the spherical to the more realistic based on boundary and finite elements. The accuracy of cortical imaging and source localization is affected by the modeling errors in head model and conductivity, approximation of the forward and inverse problems by linear equations, noise, and reference electrode.

Compared to the cortical imaging and source localization techniques, surface Laplacian is a simpler, reference-independent, and model-independent technique (Hjorth 1975; Katznelson 1981; Oostendrop and Oosterom 1996). The surface Laplacian is proportional to the radial source intensity and therefore is an estimate of the radial current source density. The superiority of the surface Laplacian over that of the potential has been reported in many applications (Srinivasan et al. 1998; Tenke et al. 1998; Edlinger et al. 1997; van Burik et al. 1998). Edlinger et al. (1997) studied sensory, motor and cognitive events and reported that all methods (surface Laplacian, cortical imaging, source localization) yield similar results.

There are two general approaches for computing surface Laplacian. The local approach computes the surface Laplacian at a scalp location based on the measured potentials at neighboring electrodes. These methods range from the finite difference scheme (Hjorth 1975; Katznelson 1981) to local modeling of the scalp and the potential surface (Le

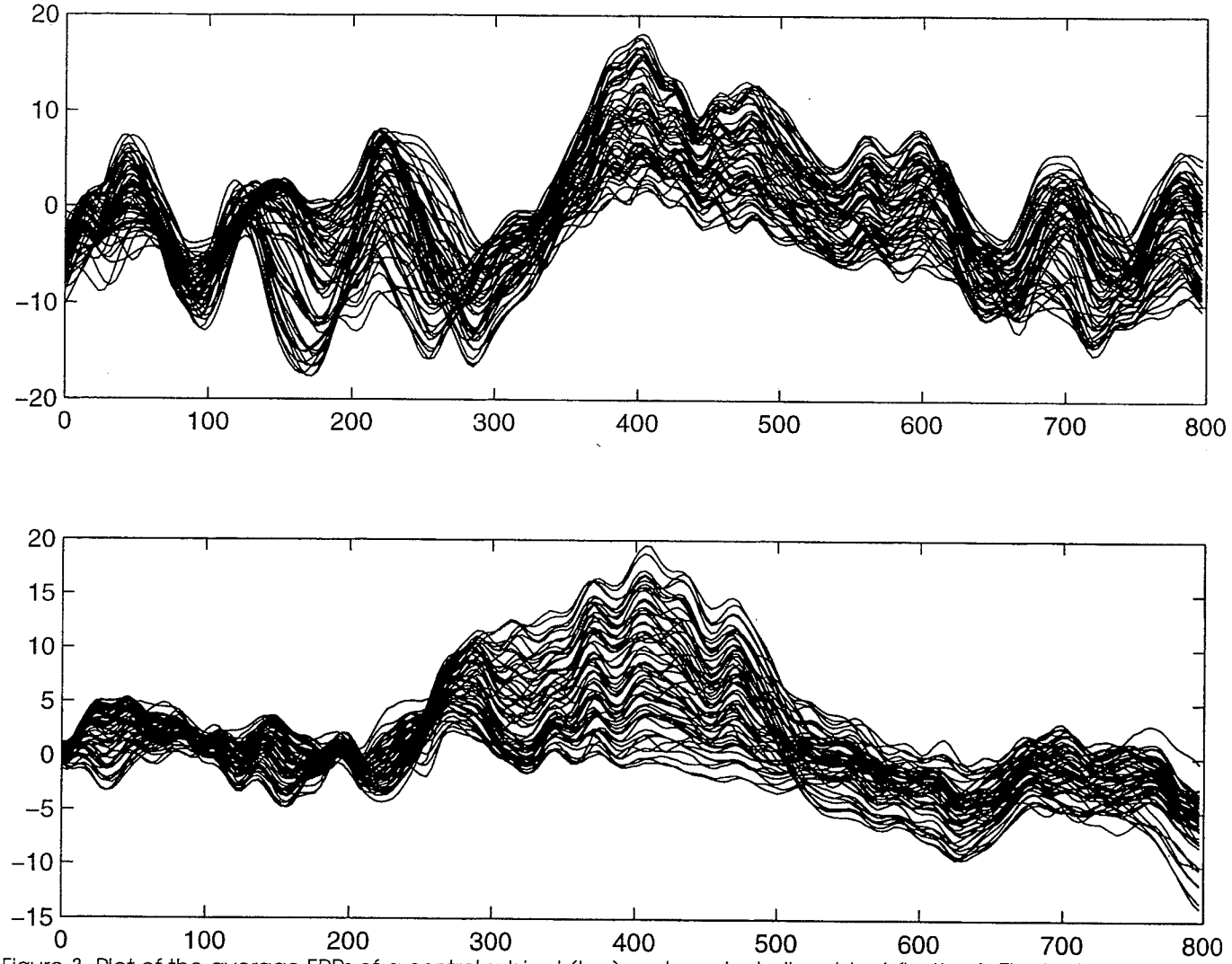

Figure 3. Plot of the average ERPs of a control subject (top) and an alcoholic subject (bottom). The horizontal unit is in msec and the vertical unit is in microvolts.

et al. 1994). The global approach constructs an analytic potential surface based on the potentials from all electrodes. The popular approach is based on the plate or spherical splines (Perrin at al. 1987; Perrin at al. 1989; Babiloni et al. 1996; Srinivasan et al. 1998; Gevins 1996) because the spline model provides a means of obtaining both potential maps and the surface Laplacian estimation. To use spherical harmonics to represent scalp potentials, one often assumes a three or four concentric spherical head model (Srinivasan et al. 1998; Srinivasan et al. 1996; Tenke et al. 1998; Babiloni et al. 1998).

The surface Laplacian is very sensitive to higher spatial frequencies and noise (Le et al. 1994; Srinivasan et al. 1998; Babiloni et al. 1998; Babiloni et al. 1995). To construct a robust algorithm, low-pass spatial filters are employed to obtain regularized (smooth) potential surface and surface Laplacian (Le et al. 1994; Babiloni et al. 1998; Babiloni et al. 1995). Linear low-pass spatial filters are weighted averaging schemes. If more neighboring potentials are averaged to estimate the potential at a scalp location, the noise will be greatly reduced in the estimate. But the bias will be increased because the potential surface is not flat. On the other hand, if few neighboring potentials are averaged to estimate the potential at a scalp location, the bias will be small but the noise will not be reduced by much. Several methods have been proposed to determine how much filtering should be applied to the recorded potentials. Babiloni et al. (1998) reported that λ correction and Tikhonov regularization provide more precise Laplacian solutions than the generalized cross-validation computation. Increasing the number of electrodes will dramatically improve the estimate of the surface Laplacian by a local method. On one hand, there are many neighboring electrodes and the weighted aver-

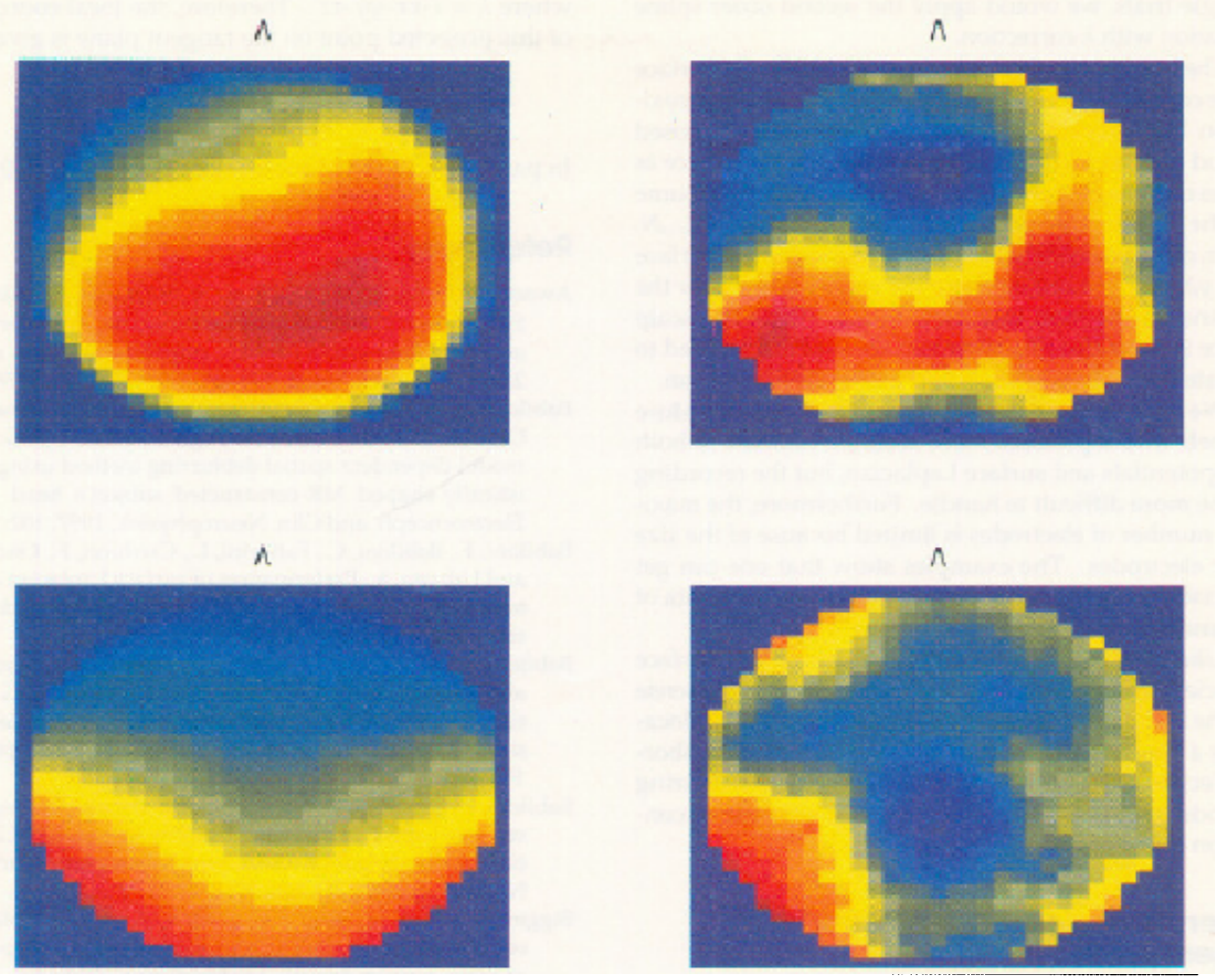

Figure 4. Plot of the estimated potentials (left column) and surface Laplacian (right column) at 400 msec after target stimulus. The top row is for the control subject and bottom row for the alcoholic subject. The maps are plotted from the view of top head.

aging of the potentials from these electrodes greatly reduces the noise. On the other hand, these neighboring electrodes are close to each other and the potentials at these electrodes are about the same as the potential at the scalp location where surface Laplacian is being estimated, due to the volume conductivity of the head. The weighted average gives a good estimate of the potential at the scalp location. Therefore, local methods benefits most from increasing the number of the electrodes.

Another problem for the previous local methods is that surface Laplacian at peripheral electrodes can not be estimated because some neighboring electrodes do not exist (Le et al. 1994). Even though the global approach sometimes produces a distorted Laplacian estimate (Biggins et al. 1991; Fein et al. 1991), Babiloni et al. (1995) reported that the best surface Laplacian estimates were computed by second order spline including λ correction. Since Laplacian is a local property of the potential surface, one can expect that a properly implemented local method would be better.

Conclusion

We have implemented a local method for estimating brain potentials and surface Laplacian, based on the assumption that the potential surface is smooth due to the volume conduction of the brain. The implementation fulfills the four requirements in the introduction. Compared to the second order spline estimation with λ correction, which is the best among previous implementations (Babiloni et al. 1995), this method is better for data with high SNR, competitive for normal SNR, and poor for small SNR. Therefore, we would apply this method to cleaner data such as the averaged ERP. For very noisy data such

as single trials, we would apply the second order spline estimation with λ correction.

The head is assumed to have a spherical scalp surface for the convenience of the simulation. It is a good approximation but it is not a realistic model. The proposed method can be used to estimate a realistic scalp surface as long as one has the coordinates of the electrodes. Assume that the N electrode coordinates are (x_i, y_i, z_i), $i=1,...,N$. Then z_i can be considered as the measurement of a surface at (x_i, y_i), and one can estimate the scalp surface over the xy plane by the local quadratic fitting. Once the scalp surface is estimated, the proposed method can be used to estimate the scalp potentials and the surface Laplacian.

We have considered data using 61 channels. More channels would produce more accurate estimate of both scalp potentials and surface Laplacian, but the recording may be more difficult to handle. Furthermore, the maximum number of electrodes is limited because of the size of the electrodes. The examples show that one can get reasonable estimate of the surface Laplacian from data of 61 channels.

Like other implementations for estimating surface Laplacian, this method is also a linear method in the sense that the estimate of the surface Laplacian at a scalp location is a weighted average of the potentials at neighboring electrodes. The accuracy of such linear debluring methods is limited because of the nonlinear volume conduction in the brain.

Appendix: Projection of electrodes on tangent plane

The upper half of the scalp surface can be parametrized as (x, y, z), where

$$z = s(x,y) = \sqrt{1-x^2-y^2}, \quad |x| \le 1, \quad |y| \le 1, \quad x^2+y^2 \le 1.$$

The center of the brain is parametrized as (0,0,0). At a scalp location (x,y,z), the normal vector of the scalp surface is also (x,y,z). The two vectors C_1 and C_2, defined by

$$C_1 = \left(-xz,-yz,1-z^2\right)/\sqrt{1-z^2}, \quad C_2 = (y,-x,0)/\sqrt{1-z^2},$$

if $x^2 + y^2 > 0$, form an orthogonal coordinate system on the tangent plane at (x,y,z). If $x^2 + y^2 = 0$, a natural coordinate system on the tangent plane at (0,0,1) is formed by two vectors C_1=(1,0,0) and C_2=(0,1,0).

For any point (x', y', z') $(z' = s(x', y'))$ on the scalp surface, its projection on the tangent plane at (x,y,z) is given by

$$(x'_p, y'_p, z'_p) = (x, y, z) + \lambda(x', y', z')$$

where $\lambda = 1-xx'-yy'-zz'$. Therefore, the local coordinates of this projected point on the tangent plane is given by

$$(u', v') = (\lambda\, C_1(x', y', z')^t, \lambda C_2(x', y', z')^t).$$

In particular, the local coordinates of (x,y,z) is (0,0).

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