Wavelet Based Multigrid Reconstruction Algorithm for Optical Tomography

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A perturbation approach has been previously presented for the inverse problem in optical tomography [3]. To solve the linear perturbation equation, several iterative algorithms have been developed, including projection onto convex sets (POCS), CGD, multigrid reconstruction [1], layer stripping [2],[5] (for TR data), Regularized least squares (RLS) [5] [4], and total least squares (TLS) [6]. One challenging problem in solving the perturbation equation is that the computation complexity is usually very high due to the extremely large dimension of the weight matrix. In order to reduce the computation time, in this paper, we propose a wavelet based multiresolution reconstruction algorithm for solving the perturbation equation.

Let the perturbation equation be represented by $\mathbf{y} = \mathbf{Hx}$ and let the transform matrices for $\mathbf{x}$ and $\mathbf{y}$ be represented by $\mathbf{W_x}$ and $\mathbf{W_y}$, respectively, multiplying $\mathbf{y} = \mathbf{Hx}$ from left by $\mathbf{W_y}$ and inserting $\mathbf{W_x}^T\mathbf{W_x} = \mathbf{I}$ in between $\mathbf{H}$ and $\mathbf{x}$, we obtain:

$$\widetilde{\mathbf{H}}\,\widetilde{\mathbf{x}} = \widetilde{\mathbf{y}}, \tag{1}$$

where $\widetilde{\mathbf{H}} = \mathbf{W_y H W_x}^T$, $\widetilde{\mathbf{y}} = \mathbf{W_y y}$ and $\widetilde{\mathbf{x}} = \mathbf{W_x x}$. Here we assume the transform matrix is orthogonal so that $\mathbf{W_y}^T\mathbf{W_y} = \mathbf{I}$. Eq. (1) is the perturbation equation in the wavelet domain. By exploiting the multiresolution property of wavelet transform, multigrid method is used to reduce the computational time. Specifically, in a two grid implementation, let $\widetilde{\mathbf{x}} = [\widetilde{\mathbf{x}}_l, \widetilde{\mathbf{x}}_h]^T$, $\widetilde{\mathbf{y}} = [\widetilde{\mathbf{y}}_l, \widetilde{\mathbf{y}}_h]^T$, and

$$\widetilde{\mathbf{H}} = \begin{bmatrix} \widetilde{\mathbf{H}}_{ll} & \widetilde{\mathbf{H}}_{lh} \\ \widetilde{\mathbf{H}}_{hl} & \widetilde{\mathbf{H}}_{hh} \end{bmatrix},$$

where the subscript l represents low frequency, subscript h represents high frequency, and subscripts ll, lh, hl and hh represent LL, LH, HL and HH frequency band in wavelet decomposition, respectively. We first obtain the course grid solution by solving

$$\widetilde{\mathbf{H}}_l\,\widetilde{\mathbf{x}}_l = \widetilde{\mathbf{y}}_l\,. \tag{2}$$

This solution can be obtained very fast since the dimension of the system is reduced by half. Starting from this coarse resolution, we can obtain the full resolution solution by solving the original equation (1). Alternatively, one could choose to solve the full resolution solution for a region of interests (ROI) identified from the coarse solution.

To solve the equation in a given grid $\widetilde{\mathbf{H}}_l\,\widetilde{\mathbf{x}}_l = \widetilde{\mathbf{y}}_l$, we have investigated two approaches: the regularized least squares (RLS) and the total least squares (TLS). The RLS solution is obtained by minimizing the following error formulation:

$$E(\tilde{\mathbf{x}}_l) = \|\tilde{\mathbf{H}}_l\tilde{\mathbf{x}}_l - \tilde{\mathbf{y}}_l\|^2 + \lambda_l\|\tilde{\mathbf{x}}_l\|^2, \tag{3}$$

where λ_l is a regularization parameter at the coarse resolution.

The TLS solution is obtained by solving the Rayleigh quotient equation given by:

$$Minimize\;\; F(\widetilde{\mathbf{q}}_l) = \frac{\widetilde{\mathbf{q}}_l^T\,\widetilde{\mathbf{A}}_l^T\,\widetilde{\mathbf{A}}_l\widetilde{\mathbf{q}}_l}{\widetilde{\mathbf{q}}_l^T\,\widetilde{\mathbf{q}}_l}, \tag{4}$$

where

$$\widetilde{\mathbf{A}}_l = [\widetilde{\mathbf{H}}_l|\widetilde{\mathbf{y}}_l] \tag{5}$$

and

$$\tilde{\mathbf{q}}_l = \begin{pmatrix} \tilde{\mathbf{x}}_l \\ -1 \end{pmatrix}.$$

In the first case, the minimization is accomplished by conjugate gradient descent (CGD) method while in the second case, it is implemented by conjugate gradient (CG) method[6].

Simulation results using the RLS method in a two grid wavelet representation are shown in Fig.1.

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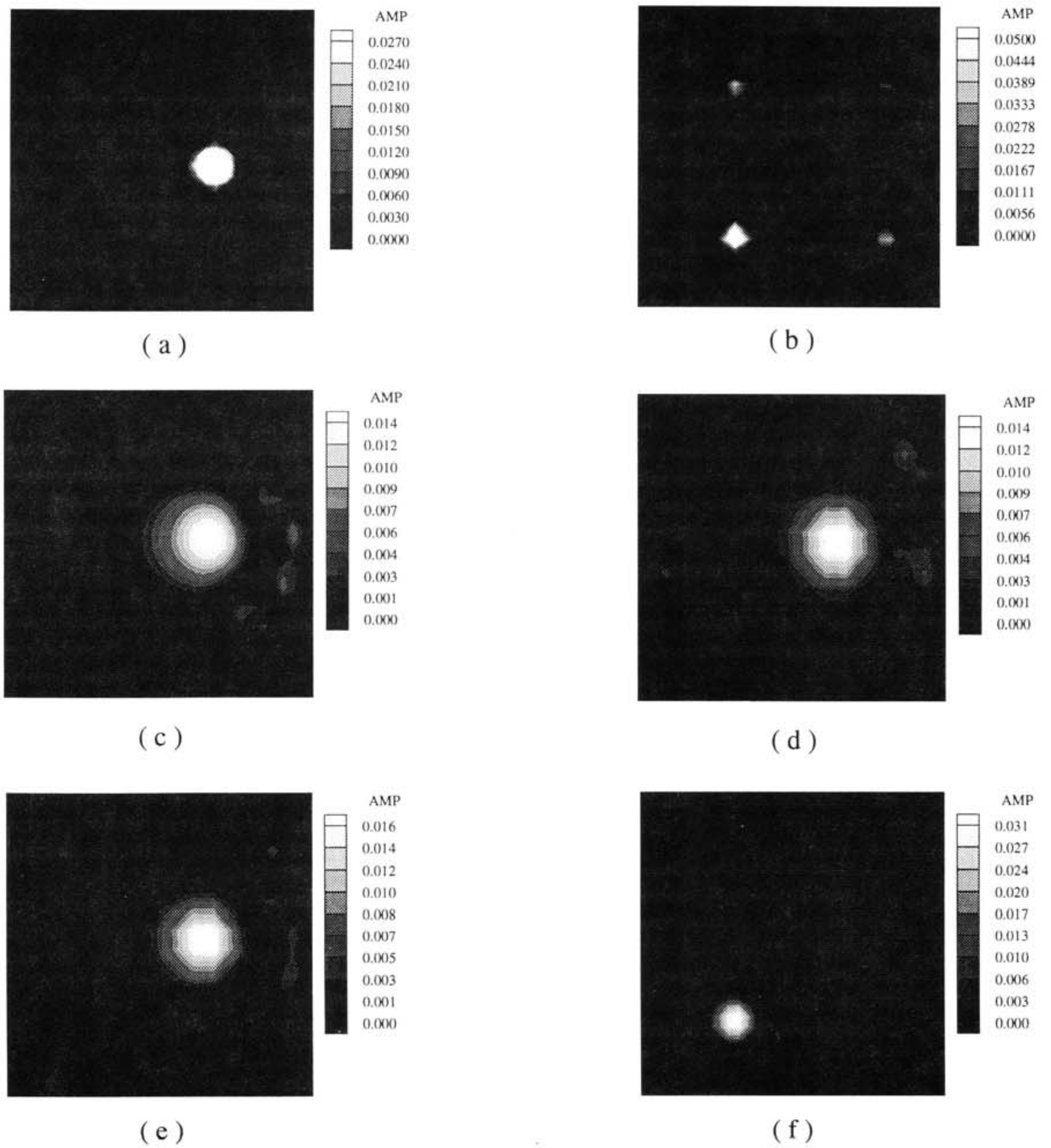

Figure 1: Reconstruction results and wavelet analysis of a medium with an off-center rod with Sin-like distribution. (a) is the original image; (b) shows the wavelet transform of original image in (a); (c) is the reconstruction result using one grid with 235 iterations; (d) is the reconstruction image using two-grids algorithm with 500 iterations in the coarse grid; (e) is the reconstruction image using two-grid algorithm with additional 200 iterations in the fine grid; and (f) is the wavelet transform of (e). The total computation time for (e) and (c) are roughly the same. The time for (d) is about 1/7 of (c).